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Evaluation of transverse thermal diffusivity of unidirectional fiber-reinforced composites

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Abstract—The applicability of the homogeneous medium approximation to transverse transient heat conduction in unidirectional fiber-reinforced composites is examined. The study focuses on the problem of transient heat conduction in thin cross-sections, encountered in applications such as the manufacture of commercial prepreg tapes, the filament winding process, etc. The flash experiment for measuring the apparent transient diffusivity was numerically simulated for a wide range of composite parameters. Based on the parametric studies, a critical sample thickness is proposed, above which the composite may be analyzed in a simplified manner as a homogeneous medium having an equivalent transient thermal diffusivity. Below the critical thickness, the homogeneous medium approximation may introduce non-negligible errors. An analytical means for the evaluation of the homogenized transient diffusivity in practical situations is also presented.

INTRODUCTION

The steady state thermal conductivity and the transient thermal diffusivity are important parameters involved in the manufacture of composite materials, and in their design for various high temperature applications. During the analysis of composite materials, it is desirable to approximate the heterogeneous material as a homogeneous medium having 'effective' properties, since this allows for substantial simplification of the process simulations. This approximation, referred to as homogenization of the composite material, is well established and widely accepted in the case of steady state heat conduction in composite media. The literature abounds with studies on the steady state heat conduction in composite materials, aimed at obtaining the 'effective' thermal conductivity, k_e, as a function of the composite properties [1–3].

On the other hand, the thermal diffusivity (α), strictly speaking, is not a characteristic property of composite materials. This is due to the fact that the unsteady heat conduction equation in which α appears as a physical constant is valid only for homogeneous media. Nevertheless, in practice, diffusivity techniques have been successfully applied to many composite materials. For example, Truong and Zinsmeister [4] studied thermal wave propagation in a layered composite material, with the waves running parallel to the layers, and proposed that such materials can be best characterized by two 'diffusivities', one representing the wave attenuation and the other describing the phase shift.

Among all the existing experimental methods of determining the diffusivity of composite materials, the notably simple and popular one is the flash method of Parker *et al.* [5], originally proposed for homogeneous materials. The method has been extended to the measurement of 'effective' thermal diffusivities of laminates [6], particulate composites [7, 8], and fiber-reinforced composites [9–11]. In the flash method, a high intensity, short duration energy pulse is imposed uniformly on the front face of a test composite specimen. Under adiabatic conditions, except for the initial pulse, the *average* rear face temperature rise is monitored and recorded as a function of time. The 'effective' thermal diffusivity, α_e, can be obtained from the 'half-time', $t_{1/2}$, which is the time required for the rear face to achieve one-half its maximum temperature rise, using the relation [5]

$$\alpha_e = \frac{1.38L^2}{\pi^2 t_{1/2}}, \tag{1}$$

where L is the specimen thickness.

Among the theoretical means of estimating the transient diffusivity of composites (such as using the concept of thermal effusivities [9]), the simplest one is the 'static' diffusivity approximation. This approximation treats the composite material as a homogeneous medium whose 'static' thermal diffusivity, α_s, is defined based on the effective thermal conductivity k_e (from a steady state heat conduction analysis [1–3]), and the effective volumetric specific heat $(\rho c)_e$ (which is a volume average of the volumetric specific heats of the constituent phases) as

$$\alpha_s = \frac{k_e}{(\rho c)_e}. \tag{2}$$

In the case of fiber-reinforced composites, the effective

NOMENCLATURE

D	fiber diameter [m]
k	thermal conductivity [W m^{-1} K^{-1}]
$\bar{k}$	dimensionless thermal conductivity, equation (8)
l_{uc}	unit cell height (Fig. 2) [m]
L	thickness of the composite sample [m]
n	index for summation
Q	energy pulse per unit area [J m^{-2}]
t	time [s]
T	temperature [K]
T_0	initial temperature of the composite sample [K]
T_M	maximum steady state temperature attained by the composite sample [K]
v	fiber volume fraction
w_{uc}	unit cell width (Fig. 2) [m]
W	width of the composite sample [m]
x, y	coordinate axes
Δx	grid size in the x-direction [m]
Δy	grid size in the y-direction, flash region depth [m].

Greek symbols

α	thermal diffusivity [m^2 s^{-1}]
α_s	'static' thermal diffusivity [m^2 s^{-1}]
β	ratio of fiber conductivity to matrix conductivity
γ	fiber packing angle (Fig. 2) [degrees]
η	ratio of fiber–matrix volumetric specific heat
θ	dimensionless temperature, equation (8)
(ρc)	volumetric specific heat [J m^{-3} K^{-1}]
$\overline{\rho c}$	dimensionless volumetric specific heat, equation (8)
τ	dimensionless time, equation (8)
$\Delta\tau$	dimensionless time step.

Superscripts and subscripts

$\bar{\ }$	dimensionless length variable scaled with respect to the sample thickness, L
1/2	'half-time' from the flash experiment
e	effective value
f	fiber
F	flash region
m	matrix
rear	rear face.

volumetric specific heat, $(\rho c)_e$, is defined in terms of the fiber and the matrix volumetric specific heats, $(\rho c)_f$ and $(\rho c)_m$ respectively, and the fiber volume fraction, v, as

$$(\rho c)_e = v(\rho c)_f + (1-v)(\rho c)_m. \quad (3)$$

It should be noted that the 'effective' and 'static' diffusivities are both fictitious diffusivities, and are only approximate characterizations of the transient thermal behavior. If an exact transient temperature distribution in the composite is desired, one has to resort to a full numerical solution. The 'effective' diffusivity is based on the 'half-time' of the temperature rise history, and does not characterize the total transient process. On the other hand, the homogenized 'static' diffusivity depends purely on the steady state properties. In comparison, and as will be shown in a later section, the 'effective' diffusivity is a more realistic characterization; however, the 'static' diffusivity is easier to evaluate, using the results of the steady state analysis [1–3]. Therefore, it is of great practical value to determine the conditions under which the 'static' diffusivity may be used.

Transverse thermal conduction in fibrous composites bears important relevance to the analysis of composite forming processes such as pultrusion [12], autoclave curing [13] and filament winding [14]. In practical applications, fiber-reinforced composite thicknesses typically vary from about 100 μm to over 1 in. [15], and the fiber diameters are normally in the range 10 μm to about 50 μm [16]. For very thin samples, the scale of heterogeneity (which in the case of fibrous composites may be regarded as the fiber diameter) is of the order of the sample thickness. Consequently, the homogeneity approximation is likely to be inaccurate (for different properties of the constituent phases), and the two diffusivities differ widely. As the thickness increases, the specimen approaches homogeneity and the 'effective' diffusivity approaches the 'static' diffusivity. A comparison between the homogenized 'static' diffusivity (equation (2)) and the experimentally measured 'effective' thermal diffusivity (equation (1)) yields a criterion for the applicability of homogenization.

The most relevant investigations in the literature concerning transient heat conduction in fiber-reinforced composites are those of Taylor and coworkers [10, 11]. Taylor and Kelsic [10] studied coarse-weave fiber-reinforced composites with fiber volume fractions less than 30%. Their primary concern, however, was heat flow along fibers at least partially aligned in the direction of heat flow. They used the flash technique [5] to assess the influence of the fiber-to-matrix conductivity, β, the fiber volume fraction, v, and the fiber orientation with respect to the direction of heat flow on the 'effective' thermal diffusivity. Based on their results, they concluded that the conductivity ratio, β, is the most important parameter affecting the transient, thermal behavior of fibrous composites, with the fibers at least partially oriented along the heat flow direction.

Taylor *et al.* [11] examined the applicability of the flash technique [5] for measuring thermal diffusivity of fine-weave fiber-reinforced 3-D carbon/carbon composites. It was qualitatively concluded that the material may be treated as homogeneous for the purposes of transient heat conduction if the sample thickness is much greater than the 'heterogeneity dimension'.

The present work addresses the limitations of using the homogenized 'static' transverse diffusivity in practical situations, where the fiber volume fraction ranges from near 30% to about 70%, and the fibers are relatively randomly distributed in the matrix. While the use of the 'static' diffusivity in thick cross-sections is obvious, the applicability of homogenization to thin specimen needs to be justified. With this objective, the focus of the paper is on transient heat conduction in thin samples.

Transient heat conduction in thin cross-sections, transverse to the fibers, is frequently encountered in the manufacture of commercial prepregs, and in the analysis of filament winding processes [14], where the thicknesses are about 100–300 μm and the fiber diameters about 15–30 μm. It is the intent of the present study to devise a *quantitative* criterion which will enable the use of the homogenized 'static' diffusivity with confidence in the practical range of composite parameters. To the authors' knowledge, such a criterion for transient transverse conduction in fiber-reinforced composites does not exist in the literature. The numerical simulation of the flash experiment [5] for a wide range of composite parameters is described. The 'effective' diffusivity obtained from the simulation is verified to be a realistic measure of the transient thermal behavior of fibrous composites, and has been used as the reference for comparison with the homogenized 'static' diffusivity. An effective analytical method for estimating the homogenized 'static' diffusivity of practical composites, having ordered or disordered fiber arrangements, is also presented.

ANALYSIS

Before presenting the simulation details of the flash experiment, the initial and final conditions on the temperature of the test specimen are identified.

Figure 1 shows a schematic of the flash experiment [5] and the cross-section of a general unidirectional fiber-reinforced composite sample. In the flash experiment, the front face of a thermally insulated composite sample of thickness L and initial temperature T_0, is uniformly irradiated with a short duration energy pulse of magnitude Q per unit front surface area. At time zero, the energy pulse heats up a small flash depth Δy from the front face, to a flash temperature T_F, as per the relation

$$(\rho c)_e \Delta y (T_F - T_0) = Q \tag{4}$$

where $(\rho c)_e$ is the 'effective' volumetric specific heat of the sample, defined in equation (3).

Since the sample is thermally insulated, the steady state maximum temperature T_M, attained by the entire sample of thickness L, is given by

$$(\rho c)_e L (T_M - T_0) = Q. \tag{5}$$

The 'half-time', $t_{1/2}$, for the sample is the time when the rear face temperature rise equals $(T_M - T_0)/2$, and the 'effective' thermal diffusivity is then evaluated by equation (1).

Formulation

The process under consideration is the one-dimensional transient conduction along the negative y-direction in Fig. 1. In the numerical simulation, one needs to analyze a representative finite width, W, as shown schematically by the dashed lines in Fig. 1, so as to account for the influence of the fibers on the heat flow. Thus the simulation consists of solving the transient heat conduction equation in a two-dimensional composite domain. Of particular interest is the rear face temperature rise as a function of time, which yields the 'half-time' for the evaluation of the 'effective' thermal diffusivity α_e. The appropriate choice of the width, W (Fig. 1), will be addressed in a later section. The generalized mathematical model for the simulation is developed here.

In a dimensionless form, the two-dimensional unsteady heat conduction equation in the sample, with the associated adiabatic boundary conditions, is

$$\overline{\rho c}\frac{\partial \theta}{\partial \tau} = \frac{\partial}{\partial \bar{x}}\left(\bar{k}\frac{\partial \theta}{\partial \bar{x}}\right) + \frac{\partial}{\partial \bar{y}}\left(\bar{k}\frac{\partial \theta}{\partial \bar{y}}\right) \tag{6}$$

$$\frac{\partial \theta}{\partial \bar{y}}(\bar{x},0) = \frac{\partial \theta}{\partial \bar{y}}(\bar{x},1) = \frac{\partial \theta}{\partial \bar{x}}(0,\bar{y}) = \frac{\partial \theta}{\partial \bar{x}}(\bar{W},\bar{y}) = 0 \tag{7}$$

where the following non-dimensional groups have been used:

$$\theta = \frac{T - T_0}{T_M - T_0};\quad \theta_F = \frac{T_F - T_0}{T_M - T_0};\quad \tau = \frac{\alpha_m t}{L^2};$$

$$\tau_{1/2} = \frac{\alpha_m t_{1/2}}{L^2};\quad \bar{x} = \frac{x}{L};\quad \bar{y} = \frac{y}{L};\quad \bar{W} = \frac{W}{L};$$

$$\Delta\bar{x} = \frac{\Delta x}{L};\quad \Delta\bar{y} = \frac{\Delta y}{L};\quad \bar{k} = \frac{k}{k_m};\quad \beta = \frac{k_f}{k_m};$$

$$\overline{\rho c} = \frac{(\rho c)}{(\rho c)_m};\quad \eta = \frac{(\rho c)_f}{(\rho c)_m}. \tag{8}$$

In equation (8), the temperature rise, θ, is normalized with respect to the steady state maximum temperature rise $(T_M - T_0)$, so that the dimensionless rear face temperature rise is between 0 and 1. Also, τ represents the dimensionless time variable, the dimensionless 'half-time', $\tau_{1/2}$, corresponds to the average rear face θ of 1/2, and all length variables are scaled with respect to the sample thickness L (denoted by overbars). The thermal conductivity, k, and the volumetric specific heat, (ρc), are normalized in terms of the respective matrix properties (subscript m). The subscript f in equation (8) denotes the properties of the fiber.

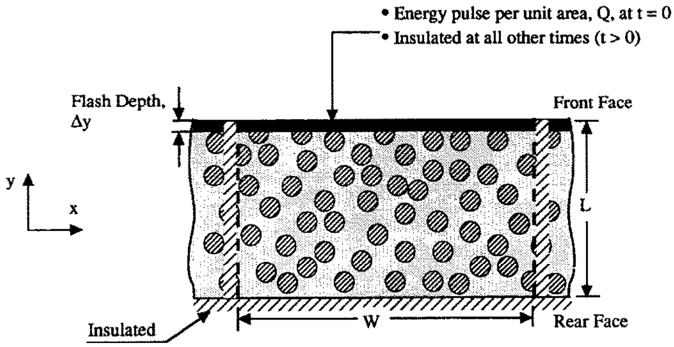


FIG. 1. Schematic of the flash experiment. A short duration energy pulse imposed on the front face initially at time = 0. Adiabatic conditions maintained for time > 0. For the numerical simulation, a finite width (shown by dashed lines) of the sample is considered.

The initial condition is the uniform dimensionless temperature, θ, = 0 throughout the sample except for the flashed region of depth $\Delta\bar{y}$, which is at the dimensionless flash temperature, θ_F. From equations (4), (5) and (8), it follows that θ_F equals $1/\Delta\bar{y}$. The initial condition may then be expressed as

$$\begin{aligned} \theta &= 0 \quad \text{for} \quad 0 \leqslant \bar{y} < (1-\Delta\bar{y}); \\ \theta &= \theta_F \quad \text{for} \quad (1-\Delta\bar{y}) \leqslant \bar{y} \leqslant 1. \end{aligned} \tag{9}$$

Parameters

A fibrous composite medium, as far as transient thermal conduction is concerned, is described in terms of the fiber–matrix thermal conductivity and volumetric specific heat ratios β and η respectively, the fiber volume fraction v, and the geometric arrangement of the fibers in the matrix. In order to systematically analyze the effect of the relative fiber arrangements, the fiber geometry was assumed to be ordered in rectangular and staggered arrays, shown schematically in Fig. 2. Also identified in Fig. 2 are the representative unit cells associated with the two arrangements. Different fiber arrangements can be generated by varying the fiber packing angle, γ. For *symmetric* ordered arrangements (rectangular and staggered arrays) as seen in Fig. 2, it suffices to analyze only one unit cell width of the cross-section, since the behavior is identical in every unit cell width. Consequently, $\bar{W}$ in equation (7) equals $\bar{w}_{uc}$, which is the unit cell width, w_{uc} (Fig. 2), scaled with respect to the sample thickness, L.

The effect of the sample thickness, L, was assessed by considering four different thicknesses, which were integral multiples of the unit cell height l_{uc} (shown in Fig. 2). The range of L was selected based on preliminary studies with thicknesses up to 10 unit cell heights, which showed that the thickness was not an important parameter beyond 6 unit cell heights. Table 1 lists all the parameters and their values used in our studies. The range of values are those usually encountered in most practical applications. A total of about 600 cases was studied.

Numerical method

The governing equation, equation (6), and the associated conditions, equations (7) and (9), were solved using an Alternating Direction Implicit (ADI) finite difference scheme [17]. The two-dimensional domain $0 \leqslant \bar{x} \leqslant \bar{w}_{uc}$ and $0 \leqslant \bar{y} \leqslant 1$, representing the transformed composite cross-section, was discretized using 41 grid points along the width ($\bar{x}$), and 111 grid points along the thickness ($\bar{y}$). This mesh size was

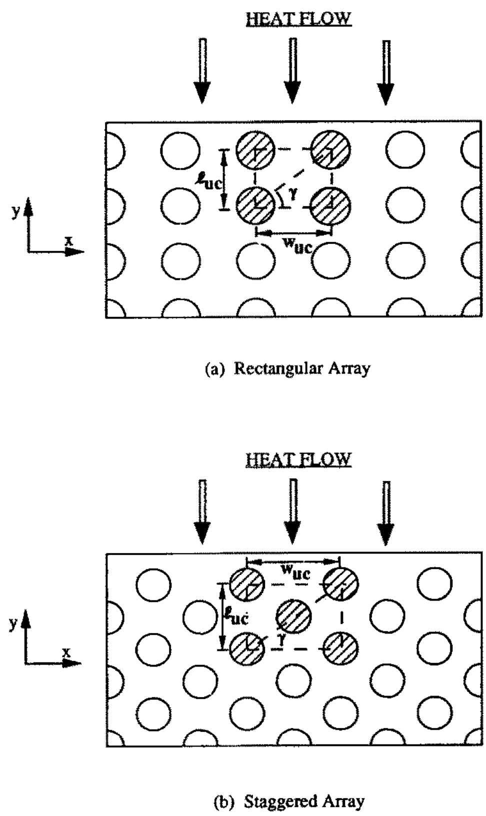


FIG. 2. Schematic of ordered fiber arrangements. (a) Rectangular array. (b) Staggered array.

Table 1. The parameters and their values used in the simulation

L	β	η	v (%)	Fiber arrangement	γ (degrees)
1, 2, 4,	0.1, 1,	0.1, 1,	30	Rectangular	30, 45, 60
6, 10	10, 100	10, 100		Staggered	25, 35, 45, 55, 65
unit cell			50	Rectangular	35, 45, 55
heights				Staggered	25, 45, 65
			70	Rectangular	45
				Staggered	30, 45, 60

found to be an optimal choice, since further refinements did not yield obvious improvements in the accuracy of the results.

The control volume approach for heterogeneous media, described in Patankar [17], was adopted in the finite difference formulation. In this approach, the properties (thermal conductivity and volumetric specific heat) of the computational cell surrounding a grid point are evaluated based on the fiber and matrix volume fractions within the computational cell. Note that the cell fiber volume fractions vary from one cell to another, and are different from the overall fiber volume fraction, v, in the composite. A cell fiber volume fraction of unity implies a fiber-filled computational cell, while a cell fiber volume fraction of 0 corresponds to a matrix-filled computational cell. If a computational cell contains a fiber–matrix interface, the cell fiber volume fraction lies between 0 and 1. The cell thermal conductivity is a *weighted harmonic average* of the fiber and matrix conductivities, with the cell volume fractions as the weights. The cell volumetric specific heat is a *weighted arithmetic average* of the fiber and matrix volumetric specific heats, with the cell volume fractions as the weights.

In our simulation, the depth of the initial heated region, $\Delta\bar{y}$ in equation (9), was chosen to be the same as the mesh size in the thickness ($\bar{y}$) direction. Since the dimensionless sample thickness was always unity (by virtue of the non-dimensionalization scheme employed), the value of $\Delta\bar{y}$ was 1/110 (the mesh size corresponding to 111 grid points) in all the cases. The average rear face temperature rise was obtained by arithmetically averaging the values of θ at the 41 grid points on the rear face. The dimensionless time step, $\Delta\tau$, ranged between 0.0001 and 0.001 depending upon the value of the fiber–matrix diffusivity ratio. A high value of the diffusivity ratio required a small time step and vice versa. All calculations were carried out on a DEC station 3100.

RESULTS AND DISCUSSION

Homogeneity criterion

Since the 'effective' diffusivity is used as the reference in obtaining the homogeneity criterion, its accuracy as a measure of the exact transient behavior needs to be assessed. This was done by comparing the rear face temperature rise versus time profiles for a composite sample and an equivalent homogeneous medium, whose thickness is the same as that of the composite material, and the transient thermal diffusivity equals the 'effective' diffusivity of the composite sample.

Figure 3 illustrates a typical result of the dimensionless rear face ($\bar{y} = 0$) temperature rise history comparison. In the figure, the dimensionless rear face temperature rise, θ_{rear}, is plotted along the vertical axis as a function of the dimensionless time, τ, along the horizontal axis. The solid curve in the figure is the numerical simulation result for a composite sample having an equilateral triangular fiber arrangement (staggered array with $\gamma = 60°$ in Fig. 2) with a fiber volume fraction of 50%, and a thickness of 2 unit cell heights. The values of the fiber–matrix thermal conductivity ratio, β, and the volumetric specific heat ratio, η, are 100 and 10 respectively. The dashed line in Fig. 3 represents the dimensionless rear face temperature rise history for an equivalent homogeneous medium having the 'effective' diffusivity of the composite, and a thickness equal to that of the composite sample. The dashed curve is obtained using the analytical solution of the transient heat conduction equation in a homogeneous medium subject to an initial flash pulse [5].

It may be seen from Fig. 3 that the transient response of an equivalent homogeneous medium agrees very well with the actual transient response for the composite from the numerical simulation. This suggests that the 'effective' diffusivity is indeed an accurate characterization of the transient thermal behavior of a composite material.

The homogeneity criterion can now be obtained based on comparison between the experimentally measured 'effective' diffusivity and the homogenized 'static' diffusivity. For each of the various combinations of the parameters given in Table 1, the normalized 'effective' thermal diffusivity, α_e/α_m, was determined using the following relation, which is equation (1) rewritten in dimensionless form:

$$\frac{\alpha_e}{\alpha_m} = \frac{1.38}{\pi^2 \tau_{1/2}}, \tag{10}$$

where $\tau_{1/2}$ is obtained from the numerical simulation results.

The normalized 'static' diffusivity, α_s/α_m, was evaluated using the non-dimensional form of equation (2) given below:

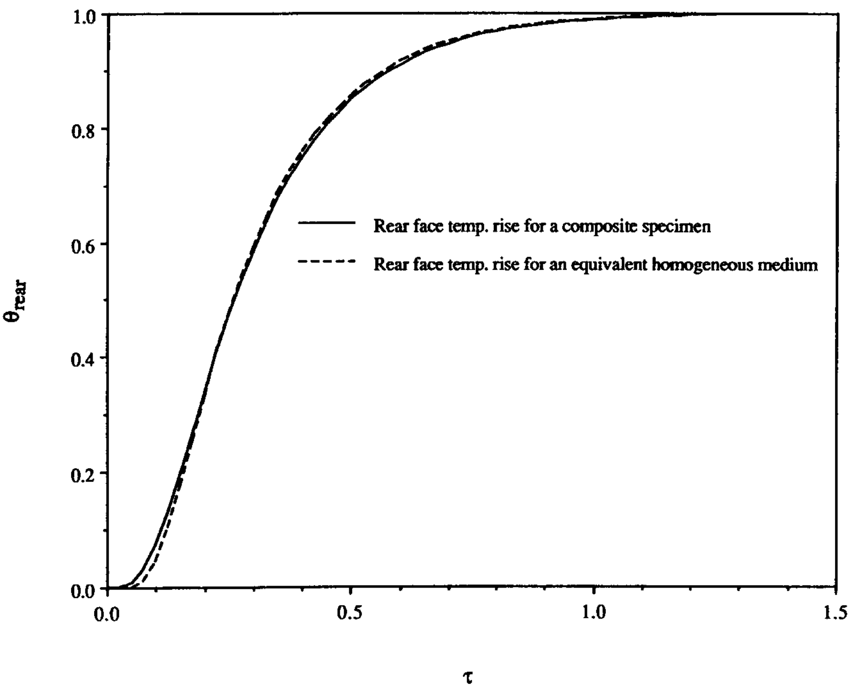


FIG. 3. Rear face temperature rise profiles for: (1) a composite sample with $L = 2l_{uc}$, $\beta = 100$, $\eta = 10$, $v = 50\%$ and equilateral triangular fiber arrangement (solid curve), and (2) a homogeneous material with $L = 2l_{uc}$ and a transient thermal diffusivity equal to α_e of the composite material (dashed curve).

$$\frac{\alpha_s}{\alpha_m} = \frac{k_e/k_m}{1+(\eta-1)v}, \tag{11}$$

where k_e/k_m is the 'effective' thermal conductivity ratio and η is the fiber–matrix volumetric specific heat ratio defined in equation (8). The 'effective' thermal conductivity ratio was obtained by numerically solving the steady state heat conduction equation in the composite. The details of the numerical solution of the steady state equation are not presented here. The interested reader is referred to the studies in the literature such as the analysis of ordered fiber arrangements by Han and Cosner [2].

The accuracy of the homogeneous medium approximation for transient heat conduction was represented in terms of the difference between the normalized 'effective' and 'static' diffusivities, α_e/α_m (equation (10)) and α_s/α_m (equation (11)) respectively, expressed as a percentage with respect to the normalized 'static' diffusivity. Parametric studies revealed that the most important parameter affecting this percentage deviation from homogeneity is the sample thickness, L. As expected, the deviation from homogeneity varies inversely with L, thereby rendering the homogeneity approximation more accurate with increasing thickness.

Figure 4 shows frequency bar charts of the percentage deviation from homogeneity, based on all the cases studied. Shown in Fig. 4 are four plot frames corresponding to sample thicknesses, L, of l_{uc}, $2l_{uc}$, $4l_{uc}$ and $6l_{uc}$. Each of these frames is a bar plot of the percentage of the total number of cases studied (along the abscissa) which correspond to a certain percentage deviation from homogeneity (along the ordinate). The standard deviations, σ, of each of these distributions are also shown in the figure.

It may be seen from Fig. 4 that for the cases reported in Table 1, if the sample thickness is greater than 4 unit cell heights ($4l_{uc}$), the maximum deviation from homogeneity is about 10%, and in over 95% of the cases the deviation from homogeneity is less than 7.08% (2σ limits [18]). Therefore, a thickness of about $4l_{uc}$ may be regarded as 'critical', above which the deviation from homogeneity is relatively small. For thicknesses below the critical value, non-negligible errors ($>10\%$) may be introduced if homogenization were employed. Also, for sample thicknesses greater than about 2–3 unit cell heights, 2–$3l_{uc}$, the effect of all the other parameters was found to be relatively insignificant.

It is interesting to note that the above result for unidirectional fiber-reinforced composites is in agreement with the qualitative conclusions of Taylor *et al.* [19] that the concept of effective diffusivity applies to fine-weave, 3-D fiber-reinforced composites when the sample thickness exceeds *four* unit cell spacings. Their result was based on experimental studies on a fine-weave 3-D carbon/carbon composite with a nominal spacing of 0.3 in. between fiber bundles in all three directions.

The critical sample thickness for homogeneity, namely $4l_{uc}$, is valid even for disordered arrays since

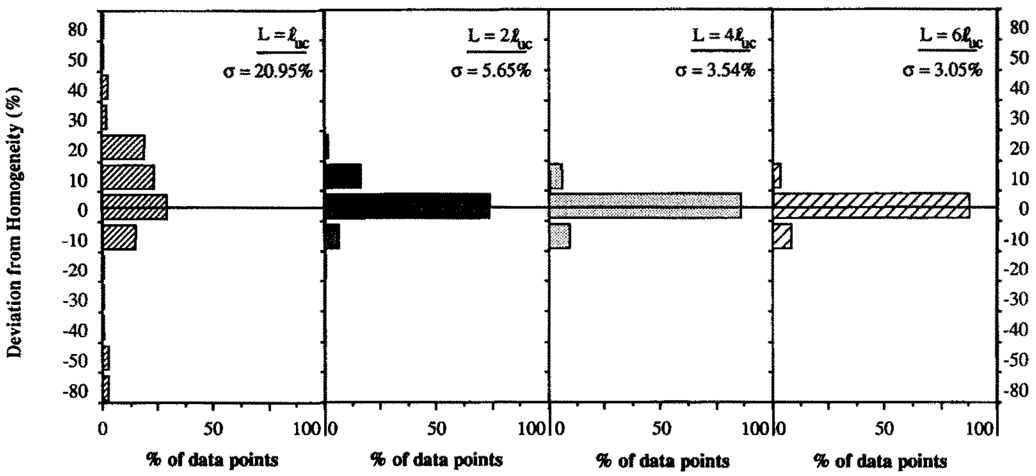
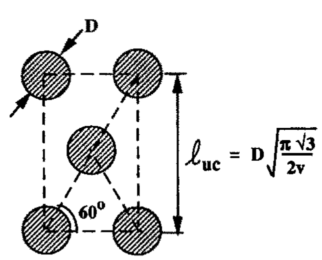


FIG. 4. Frequency bar charts of the percentage deviation from homogeneity, based on all the cases studied, for four different sample thicknesses.

the fiber arrangement is not a significant parameter beyond a thickness of $2\text{–}3l_{uc}$. However, since the term 'unit cell height' (l_{uc}) is meaningless in the context of disordered arrays, a generalized expression for the critical sample thickness is necessary.

For a given fiber volume fraction, v, the equilateral triangular arrangement (staggered array with a fiber packing angle, γ, of 60° in Fig. 2) shown in Fig. 5 may be considered representative of a uniform random fiber arrangement. (In terms of energies, hypothetically, if the fibers are assumed to mutually repel one another, this also represents the minimum potential energy configuration.) It could be argued that the square packing array (rectangular or staggered array with a fiber packing angle, γ, of 45° in Fig. 2) also approximates a uniform random fiber arrangement. However, the equilateral triangular distribution is chosen as representative since its unit cell height is larger than that of a square packing array, and consequently the resulting generalized critical thickness will be a conservative estimate.



Critical Thickness = $4(\ell_{uc}) = 4D\sqrt{\frac{\pi\sqrt{3}}{2v}}$

Homogeneity Criterion: $L \geq 4(\ell_{uc}) \Rightarrow \frac{L}{D} \geq 4\sqrt{\frac{\pi\sqrt{3}}{2v}}$

FIG. 5. Equilateral triangular fiber arrangement and the generalized homogeneity criterion.

The expressions for the unit cell height of an equilateral triangular arrangement, l_{uc}, and the corresponding critical thickness, $4l_{uc}$, are given in Fig. 5. Therefore, the homogeneity criterion may be expressed *conservatively* as

$$\frac{L}{D} \geqslant 4\sqrt{\frac{\pi\sqrt{3}}{2v}} \tag{12}$$

where D is the fiber diameter. From Fig. 4, it follows that if equation (12) is satisfied, the error introduced due to homogenization will be less than about 7% in most of the cases. If a larger error could be tolerated, the factor 4 in equation (12) may be lowered based on the percentage deviations in Fig. 4. Conversely, if a smaller error is desired, the factor 4 in equation (12) must be increased to 6 or more.

The above homogeneity criterion is valid only for transverse heat conduction in unidirectional fiber-reinforced composites. Furthermore, since the criterion is based on the parametric studies as described in Table 1, caution must be employed in the use of the criterion outside the range of parameters in Table 1.

To illustrate the application of the homogeneity criterion, equation (12), to practical situations, we consider the example of typical commercial prepregs, and typical fiber tows used in the filament winding process, which have fibers of diameter (D) about 15–30 μm, and a fiber volume fraction (v) of about 60%. For these cases, the homogeneity criterion, equation (12), requires that the thickness, L, be greater than or equal to about 130 μm and about 260 μm for $D = 15$ μm and 30 μm respectively. Typical thicknesses in industrial applications range between 100 μm and 300 μm. Therefore, considerable error would be introduced if thicknesses towards the lower end of the spectrum were analyzed using the homogenized 'static' diffusivity. For example, a 100-μm-thick sample having fibers of diameter (D) 23 μm and a fiber volume fraction (v) of 60%, assuming an equilateral triangular arrangement of fibers, corresponds to a thickness (L) to unit cell height (l_{uc}) ratio of about 2. It is evident from Fig. 4 that the deviation from homogeneity in this case could be as much as about 20%.

Estimation of homogenized diffusivity

If the homogeneity criterion, equation (12), is satisfied, the transverse thermal diffusivity may be approximated by the 'static' diffusivity (equation (11)). The accuracy of the approximation, however, depends upon the method used to evaluate k_e/k_m in equation (11). Figure 6 presents a comparison between the normalized 'static' thermal diffusivity evaluated using the numerically determined effective thermal conductivity ratio, k_e/k_m, and the normalized 'effective' diffusivity from the flash experiment, for the critical sample thickness of 4 unit cell heights. The data points in the figure correspond to the various cases studied (Table 1). The solid line diagonal to the plot frame represents the line of exact agreement, and the dashed lines are the 10% error bands. The error bands are not shown all the way down to the origin for the sake of clarity. As is evident from the figure, the agreement between the two diffusivities is extremely good in almost all the cases studied.

Since a rigorous numerical evaluation of k_e/k_m each time is impractical owing to the computational intensity, it is desirable to have an analytical expression for the normalized thermal diffusivity. With this objective, the normalized 'static' diffusivity (equation (11)) was calculated using two of the existing analytical correlations for the effective thermal conductivity ratio.

First, we utilize the simplified heat conduction model using the concept of thermal resistances in series, which is employed in some of the analyses of composite manufacturing processes [12]. 'Resistances in series' refers to the fact that the effective resistance to heat flow (which equals the inverse of the effective composite conductance) is an algebraic sum of the resistances (which equals the inverse of the conductances) due to the fibers and the matrix. The effective thermal conductivity ratio, k_e/k_m, of this model may be written as

$$\frac{k_e}{k_m} = \frac{1}{1-\left(\frac{\beta-1}{\beta}\right)v} \tag{13}$$

and the corresponding normalized 'static' diffusivity (from equation (12)) takes the following form:

$$\frac{\alpha_s}{\alpha_m} = \frac{1}{\left(1-\left(\frac{\beta-1}{\beta}\right)v\right)(1+(\eta-1)v)}. \tag{14}$$

Several improved models for the effective thermal conductivity of fiber-reinforced composites exist in the literature. Almost all the analytical models in the

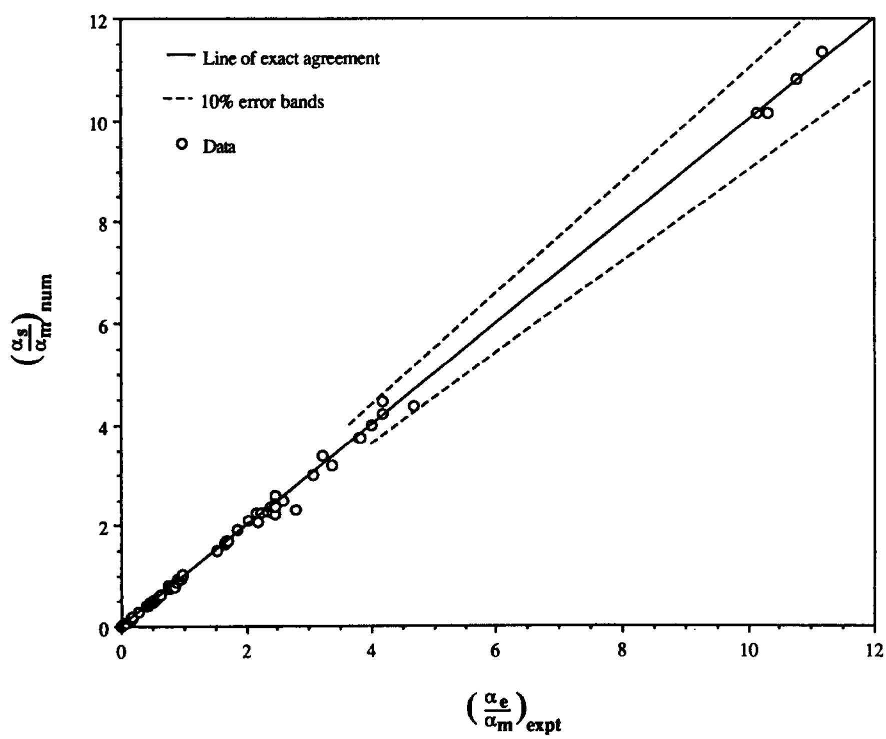


FIG. 6. Comparison between the normalized 'static' diffusivity, $(\alpha_s/\alpha_m)_{num}$, evaluated using the numerically determined effective conductivity ratios (k_e/k_m), and the normalized 'effective' diffusivity, $(\alpha_e/\alpha_m)_{expt}$, from the flash experiment simulation.

literature are applicable *either* to ordered arrangements alone, such as in ref. [1], *or* solely to disordered arrangements, such as the Milton lower bounds in ref. [20]. Recently, Pitchumani and Yao [3] proposed a generalized analytical correlation using *local fractal techniques*, which are applicable to ordered *as well as* disordered fiber arrangements. The expression for the normalized 'static' diffusivity resulting from the correlation in ref. [3] is not presented here for the sake of brevity.

Figure 7 compares the normalized 'effective' thermal diffusivity (equation (10)) obtained from the flash experiment simulation (along the abscissa), for a sample thickness of 4 unit cell heights, with the normalized 'static' diffusivities evaluated using (a) the resistances in series model (equation (14)) and (b) the analytical correlation of Pitchumani and Yao [3]. The symbols represent the various cases studied (Table 1); the solid and dashed lines are as explained in connection with Fig. 6.

It may be noted from Fig. 7 that the simulation results and the normalized 'static' diffusivity obtained using the resistances in series model (method (a) above) underpredicts considerably due to the simplified heat flow pattern assumed in the model. The correlation of Pitchumani and Yao [3] (method (b) above), on the other hand, predicts the diffusivities to within 10% of the experimentally measured values in nearly 95% of the cases considered. The expression using the correlation in ref. [3], therefore, provides quick and reasonably accurate estimates of the normalized thermal diffusivity in practical cases.

CONCLUSIONS

The limits of applicability of the homogeneous medium approximation and the use of the 'static' diffusivity for transient heat conduction in thin fiber-reinforced composites were investigated. A quantitative criterion for homogenization, based on a conservative critical sample thickness, was developed. This criterion, which is valid for transient transverse heat conduction through fibrous composites, is applicable to ordered as well as disordered fiber arrangements in the practical range of composite parameters (Table 1). The criterion, applied to commercial prepregs and fiber tows used in the filament winding process, indicates that in some cases the homogenization approximation may introduce considerable errors in the analysis.

An analytical means for the evaluation of the 'static' diffusivity (equation (14)) in practical situations (with ordered or random fiber arrangements) was proposed using the results of the steady state analysis in ref. [3]. This was shown to predict the thermal diffusivities to within 10% in almost all the cases tested.

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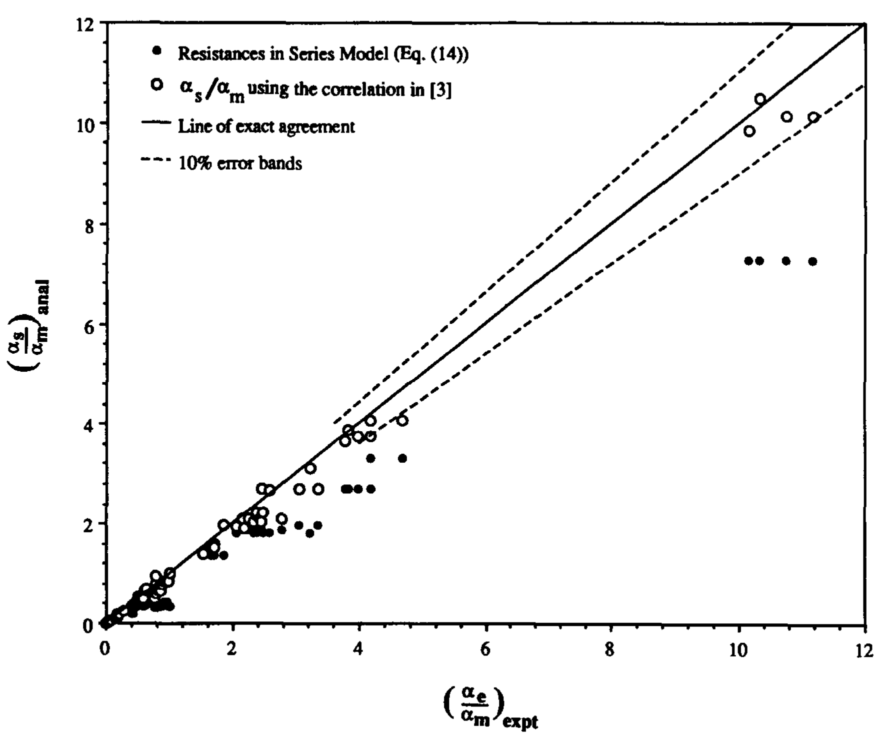

FIG. 7. Comparison between the normalized 'static' diffusivity, $(\alpha_s/\alpha_m)_{anal}$, evaluated using two of the existing analytical expressions for the effective conductivity ratio, and the normalized 'effective' diffusivity, $(\alpha_e/\alpha_m)_{expt}$, from the flash experiment simulation.

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EVALUATION DE LA DIFFUSIVITE THERMIQUE TRANSVERSALE DES COMPOSITES AVEC FIBRES UNIDIRECTIONNELLES

Résumé—On examine l'applicabilité de l'approximation d'un milieu homogène à la conduction transversale variable dans des composites renforcés unidirectionnellement par des fibres. L'étude s'attache au problème de la conduction variable dans des sections droites minces qui sont rencontrées dans des applications industrielles. La méthode flash pour mesurer la diffusivité apparente variable est numériquement simulée pour un large domaine des paramètres des composites. Basée sur des études paramétriques, on propose une épaisseur critique d'éprouvette au dessus de laquelle le composite peut être analysé de façon simple comme un milieu homogène ayant une diffusivité thermique équivalente variable. Au dessous de l'épaisseur critique, l'approximation du milieu homogène peut introduire des erreurs non négligeables. On présente aussi un moyen analytique pour l'évaluation, dans des situations pratiques, de la diffusivité variable homogénéisée.

BESTIMMUNG DER TRANSVERSALEN TEMPERATURLEITFÄHIGKEIT VON FIBERVERSTÄRKTEN KOMPOSITMATERIALIEN MIT GERICHTETEN FASERN

Zusammenfassung—Untersucht wird, inwieweit die Annahme eines homogenen Mediums bei transversaler transienter Wärmeleitung in fiberverstärkten Kompositmaterialien mit gerichteten Fasern gerechtfertigt ist. Die Studie konzentriert sich auf transiente Wärmeleitung in dünnen Querschnittsflächen, da dies in Anwendungsgebieten wie beispielsweise der kommerziellen Herstellung von 'Prepreg'-Streifen von Bedeutung ist. Die Flash-Messung zur Bestimmung der scheinbaren Temperaturleitfähigkeit wurde über einen großen Parameterbereich numerisch simuliert. Von der Parameterstudie ausgehend wird eine kritische Probendicke vorgeschlagen, oberhalb derer das Kompositmaterial vereinfacht als homogenes Material mit einer äquivalenten Temperaturleitfähigkeit betrachtet werden darf. Unterhalb der kritischen Dicke führt die Annahme eines homogenen Mediums zu nicht vernachlässigbaren Fehlern. Zusätzlich wird ein analytisches Verfahren zur praktischen Bestimmung der homogenisierten Temperaturleitfähigkeit vorgeschlagen.

ОЦЕНКА ПОПЕРЕЧНОЙ ТЕМПЕРАТУРОПРОВОДНОСТИ КОМПОЗИТОВ, АРМИРОВАННЫХ ОДНОНАПРАВЛЕННЫМ ВОЛОКНОМ

Аннотация—Исследуется применимость приближения однородной среды к поперечной нестационарной теплопроводности в композитах, армированных однонаправленным волокном. Основное внимание уделяется проблеме нестационарной теплопроводности в малых поперечных сечениях, встречающейся в таких приложениях как производство промышленных лент, броцесс намотки нити и т.д. Численно моделировался эксперимент со вспышкой для измерения кажущейся температуропроводности в широком интервале изменений параметров композитов. На основе параметрических исследований определена критическая толщина образца, выше которой композит можно рассматривать как однородную среду с эффективной температуропроводностью. В случае толщины меньше критической приближение однородной среды может вносить погрешности, которыми нельзя пренебречь. Представлен также аналитический метод оценки температуропроводности гомогенизированных сред в практических ситуациях.